

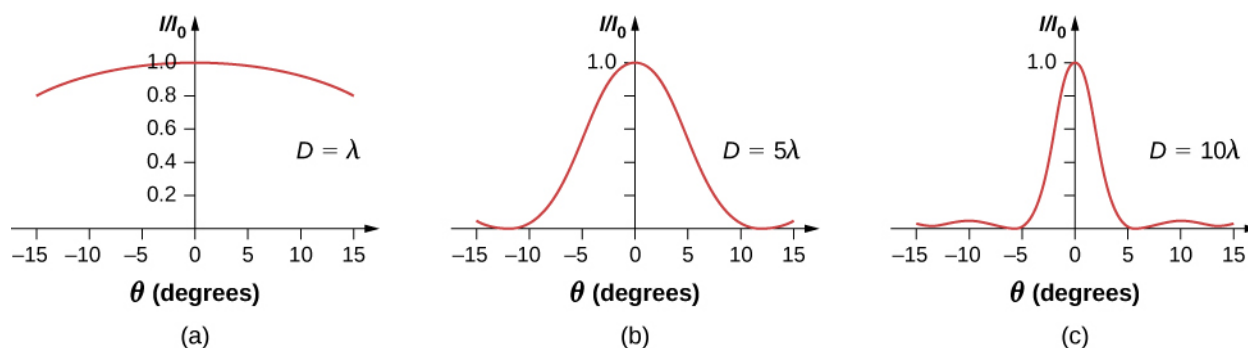


Figure 4.10 Single-slit diffraction patterns for various slit widths. As the slit width D increases from $D = \lambda$ to 5λ and then to 10λ , the width of the central peak decreases as the angles for the first minima decrease as predicted by Equation 4.1.



A diffraction experiment in optics can require a lot of preparation but **this simulation** (<https://openstaxcollege.org//21diffexpoptsi>) by Andrew Duffy offers not only a quick set up but also the ability to change the slit width instantly. Run the simulation and select “Single slit.” You can adjust the slit width and see the effect on the diffraction pattern on a screen and as a graph.

4.3 | Double-Slit Diffraction

Learning Objectives

By the end of this section, you will be able to:

- Describe the combined effect of interference and diffraction with two slits, each with finite width
- Determine the relative intensities of interference fringes within a diffraction pattern
- Identify missing orders, if any

When we studied interference in Young’s double-slit experiment, we ignored the diffraction effect in each slit. We assumed that the slits were so narrow that on the screen you saw only the interference of light from just two point sources. If the slit is smaller than the wavelength, then Figure 4.10(a) shows that there is just a spreading of light and no peaks or troughs on the screen. Therefore, it was reasonable to leave out the diffraction effect in that chapter. However, if you make the slit wider, Figure 4.10(b) and (c) show that you cannot ignore diffraction. In this section, we study the complications to the double-slit experiment that arise when you also need to take into account the diffraction effect of each slit.

To calculate the diffraction pattern for two (or any number of) slits, we need to generalize the method we just used for a single slit. That is, across each slit, we place a uniform distribution of point sources that radiate Huygens wavelets, and then we sum the wavelets from all the slits. This gives the intensity at any point on the screen. Although the details of that calculation can be complicated, the final result is quite simple:

Two-Slit Diffraction Pattern

The diffraction pattern of two slits of width D that are separated by a distance d is the interference pattern of two point sources separated by d multiplied by the diffraction pattern of a slit of width D .

In other words, the *locations* of the interference fringes are given by the equation $d \sin \theta = m\lambda$, the same as when we considered the slits to be point sources, but the *intensities* of the fringes are now reduced by diffraction effects, according to Equation 4.4. [Note that in the chapter on interference, we wrote $d \sin \theta = m\lambda$ and used the integer m to refer to interference fringes. Equation 4.1 also uses m , but this time to refer to diffraction minima. If both equations are used simultaneously, it is good practice to use a different variable (such as n) for one of these integers in order to keep them distinct.]

Interference and diffraction effects operate simultaneously and generally produce minima at different angles. This gives rise to a complicated pattern on the screen, in which some of the maxima of interference from the two slits are missing if the

maximum of the interference is in the same direction as the minimum of the diffraction. We refer to such a missing peak as a **missing order**. One example of a diffraction pattern on the screen is shown in **Figure 4.11**. The solid line with multiple peaks of various heights is the intensity observed on the screen. It is a product of the interference pattern of waves from separate slits and the diffraction of waves from within one slit.

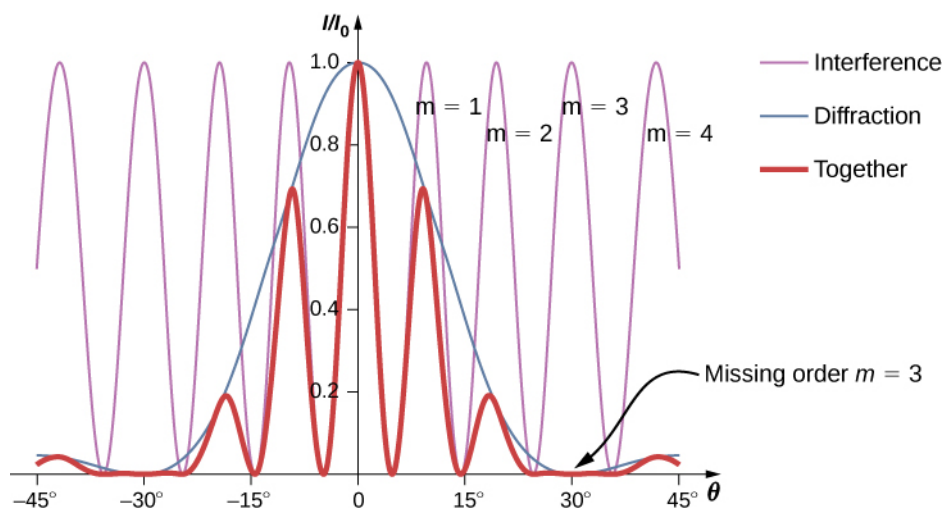


Figure 4.11 Diffraction from a double slit. The purple line with peaks of the same height are from the interference of the waves from two slits; the blue line with one big hump in the middle is the diffraction of waves from within one slit; and the thick red line is the product of the two, which is the pattern observed on the screen. The plot shows the expected result for a slit width $D = 2\lambda$ and slit separation $d = 6\lambda$. The maximum of $m = \pm 3$ order for the interference is missing because the minimum of the diffraction occurs in the same direction.

Example 4.3

Intensity of the Fringes

Figure 4.11 shows that the intensity of the fringe for $m = 3$ is zero, but what about the other fringes? Calculate the intensity for the fringe at $m = 1$ relative to I_0 , the intensity of the central peak.

Strategy

Determine the angle for the double-slit interference fringe, using the equation from **Interference**, then determine the relative intensity in that direction due to diffraction by using **Equation 4.4**.

Solution

From the chapter on interference, we know that the bright interference fringes occur at $d \sin \theta = m\lambda$, or

$$\sin \theta = \frac{m\lambda}{d}.$$

From **Equation 4.4**,

$$I = I_0 \left(\frac{\sin \beta}{\beta} \right)^2, \text{ where } \beta = \frac{\phi}{2} = \frac{\pi D \sin \theta}{\lambda}.$$

Substituting from above,

$$\beta = \frac{\pi D \sin \theta}{\lambda} = \frac{\pi D}{\lambda} \cdot \frac{m\lambda}{d} = \frac{m\pi D}{d}.$$

For $D = 2\lambda$, $d = 6\lambda$, and $m = 1$,

$$\beta = \frac{(1)\pi(2\lambda)}{(6\lambda)} = \frac{\pi}{3}.$$

Then, the intensity is

$$I = I_0 \left(\frac{\sin \beta}{\beta} \right)^2 = I_0 \left(\frac{\sin (\pi/3)}{\pi/3} \right)^2 = 0.684 I_0.$$

Significance

Note that this approach is relatively straightforward and gives a result that is almost exactly the same as the more complicated analysis using phasors to work out the intensity values of the double-slit interference (thin line in **Figure 4.11**). The phasor approach accounts for the downward slope in the diffraction intensity (blue line) so that the peak near $m = 1$ occurs at a value of θ ever so slightly smaller than we have shown here.

Example 4.4

Two-Slit Diffraction

Suppose that in Young's experiment, slits of width 0.020 mm are separated by 0.20 mm. If the slits are illuminated by monochromatic light of wavelength 500 nm, how many bright fringes are observed in the central peak of the diffraction pattern?

Solution

From **Equation 4.1**, the angular position of the first diffraction minimum is $\theta \approx \sin \theta = \frac{\lambda}{D} = \frac{5.0 \times 10^{-7} \text{ m}}{2.0 \times 10^{-5} \text{ m}} = 2.5 \times 10^{-2} \text{ rad}$.

Using $\sin \theta = m\lambda$ for $\theta = 2.5 \times 10^{-2} \text{ rad}$, we find

$$m = \frac{d \sin \theta}{\lambda} = \frac{(0.20 \text{ mm})(2.5 \times 10^{-2} \text{ rad})}{(5.0 \times 10^{-7} \text{ m})} = 10,$$

which is the maximum interference order that fits inside the central peak. We note that $m = \pm 10$ are missing orders as θ matches exactly. Accordingly, we observe bright fringes for

$$m = -9, -8, -7, -6, -5, -4, -3, -2, -1, 0, +1, +2, +3, +4, +5, +6, +7, +8, \text{ and } +9$$

for a total of 19 bright fringes.



4.3 Check Your Understanding For the experiment in **Example 4.4**, show that $m = 20$ is also a missing order.



Explore the effects of double-slit diffraction. In **this simulation** (<https://openstaxcollege.org//21doubleslitdiff>) written by Fu-Kwun Hwang, select $N = 2$ using the slider and see what happens when you control the slit width, slit separation and the wavelength. Can you make an order go “missing?”

4.4 | Diffraction Gratings

Learning Objectives

By the end of this section, you will be able to:

- Discuss the pattern obtained from diffraction gratings
- Explain diffraction grating effects